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Capital Asset Pricing Model: An Overview

Abstract

In this essay, I reviewed capital asset pricing model (CAPM), the Sharp equilibrium theory of markets under conditions of risk, and the Markowitz optimal portfolio theory. I also, discussed the empirical problems with CAPM, its modified models including, the zero beta model, single standard model, and multi factor models. Finally, I reviewed the importance of CAPM tenets and corporate business decision making.

Theoretically, the capital asset pricing model (CAPM) is a model for describing equilibrium prices in the market for assets. This model first was developed by Sharpe (1964), who won the Nobel Prize for that, Lintner (1965), and Mossin (1966) and was further elaborated by other scholars (Black, 1972). CAPM which is originally based on the optimal portfolio selection theory of Markowitz (1952), was expanded by Sharp and others to provide market equilibrium of asset prices under conditions of risk and the way that the price of an individual asset is related to the components of its risk (sharpe,1964). According to CAPM, investors would compare the discounted future expected pay-offs from a specific asset with the market value of that asset and sell, if market price is higher than the discounted amount and buy if the market price is lower than the discounted value of the pay-offs. The equilibrium price is achieved when the market price of each asset is equal to the discounted amount of expected future pay-offs of that asset and when all asset prices are at equilibrium, asset market is in equilibrium as a whole. Investors obtain the discounted future expected pay-offs from a specific asset with a metrics; that metrics is the investors' expected rate of return in the equity market. The question is how investors form their expectations of the rates of returns for different assets?

CAPM explains the expected return in terms of risk that is entailed in investing in a specific asset. CAPM fully describes the risk/return relationship and suggests that under some assumptions, there is a linear relationship between the expected rate of return of an individual asset and some measure of risks that is associated with that asset. CAPM that is essentially a simplified extension of Markowitz' model and shares its core assumption with Markowitz model that every investor in the capital market is risk averse. CAPM added a risk-free asset to the portfolio theory of Markowitz from which investors can lend or borrow at the risk free rate. CAPM concludes that all investors can choose a combination of risk-free asset and the portfolio of risky assets. An investor with average degree of risk aversion will invest all his or her funds in the optimal portfolio of risky assets. Investors with above average risk tolerance borrow at the risk-free rate and invest in the optimal portfolio of risky assets in excess of their own funds and investors with below average risk tolerance invest a portion of their funds in the portfolio of risky assets and lend (invest) the rest in the risk-free asset.

The Assumptions of CAPM

1-Since CAPM is an extension of Markowitz Portfolio Selection Model with some adjustments to be used for equilibrium asset prices in the capital market, its first and basic assumption that is derived from Markowitz model is the assumption that participants in the capital market are rational risk-averse investors. Thus, on the basis of this assumption, CAPM assumes that investors' decision to invest in an asset only depends on the expected rate of return and the risk of securities, risk is defined as the probability of actual returns being different from expected return. 2 - The investors are rational mean-variance portfolio optimizers and they select an efficient portfolio from the efficient frontier according to Markowitz model.3- This assumption which is called homogenous expectations or beliefs assumption is that all investors have similar economic view of the world, analyze securities in the same way, and they have identical estimates of probability distribution of securities' returns, expected rate of returns, expected variance and covariance of returns, and expected future cash flows for all securities.

4- The capital market is a perfectly competitive market that consists of many buyers and sellers of securities and like all perfectly competitive markets, each investor is a price taker and no individual investor can influence the asset market prices in the capital market. 5- All investors have homogenous holding period or investment horizon, a month, a year, or any other time period. 6- Investments are limited to the universe of all publicly traded financial assets, like stocks, mutual funds, and bonds and to a risk free asset. 7- There is an asset with zero risk from which all investors can lend or borrow any amount at a risk-free rate. 8- There is no transaction cost or any tax liability for the investors. Although, these assumptions are far from reality, but it does not nullify the CAPM conclusions and its implications. The validity of a theory should not be based on the validity of its assumptions but on how well the model's outcome explains what actually takes place in the real world.

The Standard Model of CAPM

According to CAPM the required return for investors is divided into two parts: The nominal risk free rate (NRFR) and the risk premium (RP). As per CAPM, NRFR is determined in the government treasury bills market and depends on the real rate of economic growth, expected inflation rate, and monetary policy parameters, as well as investors' time preference. The risk premium, on the contrary, is a required return by investors to compensate for the amount of risk in a specific security. This can be presented mathematically as:

$$E(R_e) = NRFR(n) + RP(n) = NRFR(n) + (\text{Market price of risk X amount of risk} \quad (1)$$

(Patterson, 1995, p. 28). On the basis of these assumptions and some mathematical manipulations the CAPM model arrived at the following formula for the equilibrium level of required return for a specific security:

$$E(R_j) = E(NRFR) + \beta_j E(R_m - NRFR) \quad (2)$$

Where E (R_j) is the expected return on security j, NRFR is the nominal risk free rate, □$_j$ is the beta or the systematic risk of security j, and E (R_m) is the expected return on the market. Typically, in practice estimates of an individual security's expected return and beta are calculated by taking some broad index, such as S&P 500 index, as a proxy for the market. Thus, according to equation (2) that represents the standard CAPM model (a) there is a linear relationship between the systematic risk of a security and investors' required returns from the security, the higher the systematic risk of a security the higher would be the returns investors require or expect from that security; (b) if the risk free rate and the market returns stay constant, there is no other variable affecting expected returns of a security except its own systematic risk; (c) the intercept of the linear relation between expected returns and systematic risk of any security is the risk free rate; and (d) the beta of the risk free rate is zero and the beta of the market portfolio is one.

In a nutshell, one can summarize the CAPM model as follows: (1) Investors view the risk as the probability of diversion of actual returns from expected return. (2) The total risk of investing in as asset can be divided into two components; systematic and nonsystematic risks. (3) on the assumption that the probability distribution of returns is a bell-shaped symmetric distribution, investors are only concerned with the expected or average return and the variance of returns reflecting the amount of risk in the investment. However, according to CAPM assumptions the nonsystematic risk of an asset that arises from firm-specific factors can be effectively eliminated by holding the asset within a well-diversified portfolio. The systematic risk, on the other hand, is related to the economy as a whole and depends on the factors out of control of the firms and cannot be eliminated by diversification. However, depending on the type of business that the firms are in, the systematic risks of different assets are different, the extent of which is measured by the co-movements of their returns with the return on the whole asset market (Patterson, 1995). The important result from this analysis is that if investors hold well-diversified portfolios, then, it is only systematic risk that affects investors' required returns. Thus, the expected returns on a security can be explicitly estimated by knowing its systematic risk or the risk free rate prevailing in the economy and the expected returns from the general market of risky assets; this is the infamous CAPM proposition (Patterson, 1995).

Sharpe (1964) argued that every investor in the capital market invests in portfolio of risky assets (called portfolio M or market portfolio); this portfolio contain all the risky securities that trade in the capital market. Due to the fact that the weights in portfolio M are all positive and no security is being shorted or being omitted in this portfolio, he called this portfolio the market portfolio. Furthermore, Sharpe proved mathematically that the weight of each security in the market portfolio is equal to the market value of that security divided by total market values of all securities. This is usually referred to by stating that the market portfolio is a market capitalization weighted portfolio. Finally, by arguing that the risk of an individual security for an investor is the risk contribution of that investment to the investor's portfolio, Sharpe demonstrated mathematically that the expected return of any security in excess of the risk-free rate is proportionate to the expected return of the market portfolio in excess of the risk free rate. As a result, the link between the expected excess return of an individual security and that of the market portfolio is the covariance of that security's expected return with the market expected return standardized by the variance of the market return.

The Security Market Line (SML)

In order for CAPM model to be true in a particular point in time, when the returns of securities are regressed against their betas, with the beta as independent variable and the returns as the dependent variable, the resulting regression line must have a positive slope. Such a line is called security market line (SML) in the literature and is used to predict expected returns of securities based on their betas. Based on SML definition, if the market return equals the risk free rate, then the expected return from any security should be equal to the risk free rate also. In other words, the intercept of SML with the vertical axis should be at the risk free rate. The SML also implies that if the actual return on a security, given its beta, falls above the SML, that the security is underpriced for its risk profile and vice versa (Reilly and Brown, 1997).

Security line can be formulated as:

$$E(R_j) = R_f + \beta_j [E(R_M) - R_f] \quad (3)$$

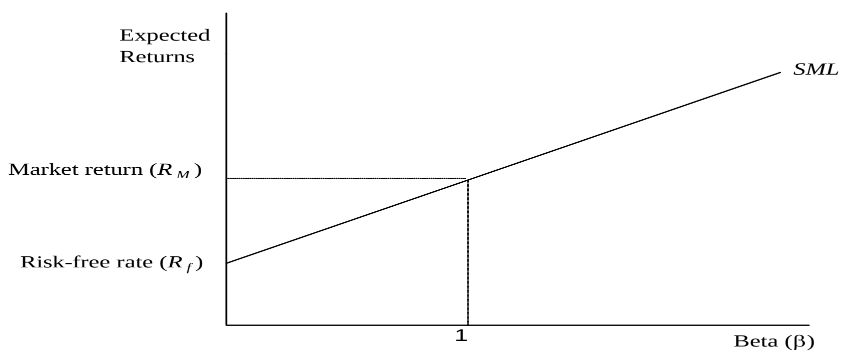

Figure 1. The security market line (SML) and the expected return-beta relationship

The Capital Market Line (CML)

Sharpe (1964) postulated that on the assumption that investors can lend and borrow unlimited amounts at a risk free rate, all investors have one optimal portfolio that is mixed with some level of lending or borrowing at the risk-free rate. This is the point of tangency between the efficient frontier and the capital allocation line (CAL) as shown in Figure (2) below; the tangent line is the capital market line (CML) and the portfolio at the point of tangency is the market portfolio or portfolio M. Therefore as it is shown in Figure (2) CML symbolizes all market portfolios M with different mixtures of lending or borrowing of the risk-free asset. An individual investor decides on an optimal portfolio of a mixture of portfolio M and the amount of lending or borrowing from risk-free asset by her/his level of risk tolerance.

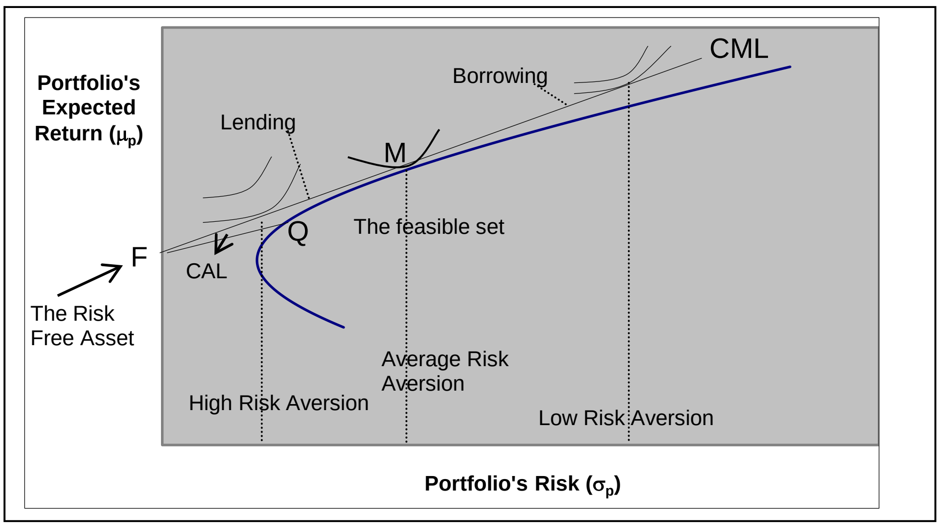

Figure 2. The capital market line (CML) and the global optimal portfolio.

Therefore, if an investor has average degree of risk tolerance, s/he will invest only in portfolio M, if s/he is high risk-averse s/he will partly invest in portfolio M and partly at the risk-free rate, and if s/he is a low risk-averse investor, s/he will invest more in portfolio M by borrowing more money at the risk-free rate. Since portfolio M encompasses all risky securities, it is called the market portfolio. Moreover, as portfolio M is diversified and consists of all risky assets, any specific risks of individual securities would have insignificant effect to the total risk of portfolio. Thus, the market portfolio does not carry any risk attached to any individual security or unsystematic risk; the risk involved in market portfolio is totally systematic caused by general macroeconomic conditions out of control by specific firm. This is the basic logic behind the popular expected return-beta relationship of CAPM that could be mathematically shown as equation 4.

$$E(R_j) - R_f = \beta_j [E(R_M) - R_f] \qquad (4)$$

Equation 4 which is the well- known CAPM expected return-beta relationship states that in equilibrium the expected risk premium for any individual security j is equal to the expected risk premium of the market portfolio times the beta of the security j, where □$_j$ is a measure of the relative risk of security j within the market portfolio.

Therefore, the excess return, or the risk premium, on the market portfolio is proportional to the variance of market portfolio, and the ratio is a measure of the average risk aversion of investors. The ratio of market excess return to market variance, is referred to as the market price of risk and plays the crucial role in derivation of the CAPM expected return-beta relationship. In fact, by estimating the expected excess return and the variance of the market portfolio from historical data, one can estimate the average degree of investors' risk aversion from a historical perspective. Change in the market risk premium ratio or the market price of risk would, therefore, imply change in the average investors' risk aversion.

Modified Models of CAPM

There are a long list of empirical irregularities that were at odds with the CAPM, and they constitute the basis of the so-called CAPM anomalies in the literature. For instance, the

CAPM does not capture any of the size, book-to-market, past return, or illiquidity effects. The existence of anomalies in the empirical testing of CAPM has led to more dynamic version of CAPM and formed the conditional CAPM or CCAPM. The modified models of CAPM can be classified into two groups: The first group approached CAPM by relaxing some of its oversimplified assumptions and the second group argued about other sources of risk that investors should compensate for and thus proposed the multi-factor CAPM.

The Zero-Beta Model

The zero-beta model modifies CAPM by dropping the assumption of the existence of risk-free asset, while keeping all other assumption of CAPM. Black (1972), who first introduced this version of CAPM, argued that with existence of inflation, there would be no such thing as a risk-free asset in the economy. Even the US Treasuries that are entirely free of default risk carry inflation risk, because they are nominal obligations and their real values would decline with inflation. Dropping the assumption of the risk-free asset availability or even putting some restrictions on the risk-free asset, such as, different rates of borrowing and lending, leads to the conclusion that the market portfolio would no longer be the optimal portfolio for all investors. Investors may select any portfolio from the efficient portfolio depending on the amount risk they choose to bear. Moreover, with investors choosing different portfolios, it is no longer obvious if the market portfolio, being the aggregate of all investors' portfolios, will even be a mean-variance efficient portfolio. This implies that if the market portfolio is no longer on the efficient frontier, then the expected return-beta relationship of CAPM will no longer reflect capital market equilibrium.

However, Black (1972) did rigorous mathematical analysis and demonstrated that the CAPM findings could be true even when there is no risk-free asset in the economy. He stated that every portfolio on the efficient frontier has a companion portfolio on the bottom part (the inefficient part) of the minimum variance portfolios with which it is uncorrelated. Because an efficient portfolio and its companion portfolio are uncorrelated, black called the companion portfolio, the zero-beta portfolio of the efficient portfolio that can be shown by Figure 3 below:

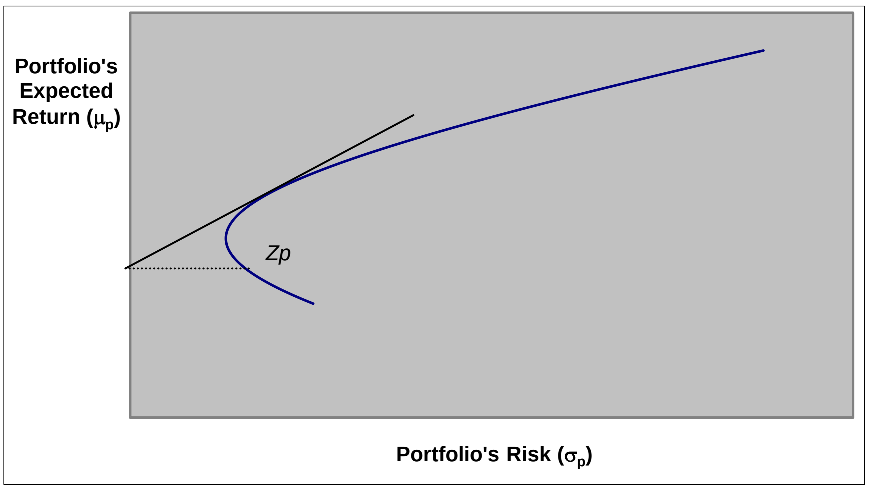

Figure3. Zero-beta companion of an efficient portfolio

Thus, Black (1972) proves that core conclusions of CAPM remain valid even if the assumption of risk-free asset is dropped from the model. Specifically, Black's model demonstrates that (a) the market portfolio is a mean-variance efficient portfolio and lies on the efficient frontier and (b) for every individual security there is an expected return-beta relationship, with the beta being the standardized covariance of the security's return with the market return. The Black model, however, keeps all other assumptions of CAPM. Further attempts by other scholars are done to verify theoretical validity of CAPM if some of the other assumptions are removed.

Single-Index Model and CAPM

The single-index model was originally suggested by Sharpe (1963) and the CAPM was subsequently developed by Sharpe (1964) as a theoretical advancement of the original version. However, later on the single-index model has gained more popularity in practice due to making less assumptions and the possibility of being tested against measurable observations. The single index model, and more generally the factor models, are based on the observation of the actual security returns and make the least assumptions about the investors' behavior or the working of the capital market. The observation that security returns move in concert implies that the same economic factors affect the outcomes of many firms. These common factors could be the business cycles, inflation, interest rates, the money supply, oil prices, or others. Thus, unexpected changes in any of these factors will create simultaneous unexpected changes in the rates of returns of the entire capital market. Because these common macroeconomic factors are interrelated, the single-index model assumes that all the relevant common factors can be grouped into one macroeconomic indicator and this single factor drives the whole security market. It is further assumed, in the single-index model, that beyond this common effect, all remaining uncertainty in the security returns is firm specific. In other words, other than the single common factor there is no other source of correlation between security returns.

With this line of reasoning, the actual or realized rate of return on any specific security j will have three components, the part expected or required by the investor as of the beginning of the holding period, the part due to unanticipated changes in the common macroeconomic factors during the holding period, and the part due to unanticipated firm specific events during the period. Mathematically, this idea can be written as:

$$R_j = E(R_j) + M_j + e_j \tag{5}$$

Where, R_j is the actual return realized on security j, $E(R_j)$ is the investors' return expectation from security j at the beginning of the holding period, M_j is the effect of unanticipated macroeconomic factors on the return of security j during the holding period, and e_j is the effect unanticipated firm specific events during the period. Now, because it is observed that different firms have different sensitivities to macroeconomic factors, M_j can be expressed as the product of the responsiveness of firm j to unanticipated macroeconomic factors and the amount of unanticipated changes in the macroeconomic factors.

Therefore, Equation 5 can be written as:

$$R_j = E(R_j) + \beta_j F + e_j \tag{6}$$

Where, F is the amount of unanticipated changes in the macro factor, and β_j is the responsiveness of security j returns to unanticipated macroeconomic factors. Equation 6 is a single-factor model of the security returns in which all the relevant economic factors are summarized into one factor. The above factor model would of no use without specifying a method to measure the factor F which is assumed to affect every security's return. The single-index model assumes that the rate of return on a broad stock market index, such as that of the S&P 500 Index, can be taken as a proxy for the common economic factor; and that is the reason for the label single-index model. Thus, the index model beta coefficient turns out to be exactly the same beta as that of the CAPM expected return-beta relationship with the difference that the theoretical market portfolio of CAPM is replaced with a well specified and observable index.

Putting it simply, it means that correlations between security returns come from a single factor; the market index factor which is the proxy for the common economic factors. This facility of the single-index model, however, is not without cost. The assumption of the single-index model that uncertainty of returns comes only from two sources, the macro and the micro sources, overlooks the fact that while certain macro factors affect all securities, there are some factors that affect many firms within the same industry without substantially affecting the broad macro economy. For example, the price of a raw material used within the computer industry affects all firms within the computer industry without much effect on the firms in other industries. Even those common macro factors that affect all firms in the economy, the magnitude of their effect is different on firms of different industries and thus aggregating all common factors into a single factor could overestimate or underestimate these differences. For example, firms in the public utility and banking industries show different responsiveness to a sudden change in interest rate then do the firms in the technology industry.

The advantage of the single index model over the CAPM model is that it arrives at similar return-risk relationship while overcoming the testability problem of CAPM. As was discussed before one central tenet or prediction of CAPM was that the market portfolio is a mean-variance efficient portfolio. To test for this one needs to build a value-weighted portfolio of enormous size that contains all the assets in the market. Such a task has not been done so far, if at all feasible, and all the tests conducted use a much smaller portfolio as a proxy for the market portfolio. Moreover, even if the tests indicate that the market proxy is an efficient portfolio, it does not prove that the actual market portfolio is efficient and thus cannot testify CAPM tenet of the efficiency of the market portfolio. An even more serious problem with CAPM is that it implies relationships among expected returns, whereas what can be observed in reality are actual or realized holding period returns that are not necessarily equal to the prior expectations. In fact this problem of ex-ante versus ex-post returns makes it impossible to test CAPM even if an actual market portfolio is constructed. To test for the efficiency of the market portfolio, one needs to show that the reward-to-variability ratio of the market portfolio is higher than that of all other portfolios. But the reward-to-variability in CAPM is set in terms of expectations, and there is no way to observe expectations directly.

This problem of measuring expectations becomes troublesome for testing the second central set of CAPM predictions, those of the expected return-beta relationship of individual securities and the resulting security market line. These relationships are expressed in terms of the expected returns of individual securities and the expected return of the market portfolio and thus one need to measure the unobservable expectations for testing them. Because of these two major problems of non-observability of the market portfolio and the expected returns CAPM the betas of individual securities cannot be empirically determined by CAPM and therefore the expected return-beta relationship for the universe of securities as exemplified in the SML cannot be tested. The single index model overcomes these shortcomings for empirical testing of CAPM by using a broad index as a proxy for the market portfolio and the average historical rates of returns of the index and of individual securities as the proxies for the expected rates of return.

Multi Factor Models and CAPM

The theory of asset pricing that started with (Sharpe, 1964; Lintner, 1965; and Black et al., 1972) was the most favored model for pricing financial assets both by the academia and practitioners. However, since the 1970s and particularly in the 1980s, researchers have recognized that this simple model does not fit well with the complexity of contemporary capital markets (Basu, 1977; Banz, 1981; Bhandari, 1988; Chan et al, 1991; Stattman, 1980. The Sharpe-Lintner-Black (SLB) model lost its credibility because of empirical contradictions, specifically because of asset pricing anomalies that was evidenced by researchers who applied the model in various stock markets across the world. On the empirical front researchers found about more factors that affect asset pricing in the capital market, some of these factors are (a) the size of the company (Banz, 1981), (b) the value effect (Chan et al., 1991), (c) price to earnings ratio effect (Basu's, 1977), (d) the relation between leverage and average return on stocks (Bhandari, 1988). Moreover, there are studies that indicate the average return of stocks in the US was positively correlated to book equity to market equity ratio (Stattman, 1980).

In the middle of all these incongruities between the empirical research and the formulation of CAPM, Fama and French (1992, 1993) developed an asset pricing model (FFM) by extending the CAPM and adding two new variables to it, (a) size which was measured by market capitalization and (b) value which was measured by Book Equity to Market Equity. Fama-French (1996) documented their model as three factor model (FF3) which has been used in a lot of empirical testing in comparison with the traditional CAPM model and has found to outperform all other theoretical asset pricing. Also, Fama-French (1993, 1996) suggested the inclusion of a factor they called distress factor which implied that companies with low price-to-book equity (P/B), price-to-earning (P/E) ratios and past sales growth (PSG) have persistently low earnings and therefore should provide higher returns to the investors. The companies with high P/B and P/E ratios and PSG have persistently high earnings and therefore generally provide lower returns to the investors.

Chiarella, Dieci, He & Li, (2013) focused on heterogeneity and evolutionary behavior of investors in the financial markets and incorporated the adaptive behavior of agents with heterogeneous beliefs and established a new version of capital asset pricing model (CAPM), named evolutionary capital asset pricing model (ECAPM). The results indicated that there is a spillover from the market price of the fundamental value of one asset to the other assets when rational behavior agents switch to better-performing trading strategies. Also, they concluded that this spill-over effect is associated with high trading volumes and they found a positive correlation between, price volatility and trading volume. They also concluded that when agents increasingly switch to better performing strategies, this rational behavior of agents can lead to instability of financial markets. This framework can also explain various types of market behavior, such as the long-term swing of market prices from the fundamental prices, asset bubbles, market crashes, the stylized facts and various kinds of power law behavior observed in financial markets (Chiarella, Dieci, He & Li, 2013).

Moreover, Abdymomunova & Morleyb (2011) developed an improved version of capital asset pricing model (CAPM) called conditional CAPM. They added time variation to the original CAPM with addition of Book-to-Market (B/M) value and suggested that they got satisfactory results. The inclusion of time-varying betas could explain the portfolio returns much better than the unconditional CAPM, especially when market volatility was considered high. On the empirical side, this study evidenced that in all cases that the unconditional CAPM were rejected, application of this model has shown a better result. Levy (2012) compared the capital asset pricing model (CAPM) by Markowitz and Sharpe with the prospect theory (PT) that was developed by Kahneman and Tversky (1979, 1992)-they won the Nobel Prize in Economics in 2002 for the development of this theory. The PT theory and the more elaborated version of it, the Cumulative Prospect Theory (CPT) (Tversky and Kahneman, 1992) challenged the Expected Utilization (EU)- risk aversion assumption of the CAPM. Kahneman and Tversky (1979) suggested an alternative paradigm to EU theory. They showed that investors make their investment decisions based on change of wealth and loss aversion and thereby they maximize the expectation of an S-shaped value function, which contains a risk-seeking segment. According to the prospect theory, the behavior of actual person is different from the assumed economic rational person; thus, prospect theory contradicts the expected utility theory and specifically the classical assumptions of the CAPM, rationality of economic agents. However, prospect theory is not aimed to be an equilibrium pricing model and therefore, cannot substitute the existing expected utility model and in particular the CAPM.

The PT and CPT have become very popular in economic research and became the foundation for a new branch in economics and finance as behavioral finance and behavioral economics. Nevertheless, since CAPM has still kept its status as the most popular asset pricing model, there must be some justifications for the use of CAPM (Berk, 1997). From theoretical ground, the CAPM was challenged by behavioral economists and psychologists (BE&P). They also, considered the expected utility assumption in CAPM is invalid and favored the prospect theory as a substitute paradigm. Moreover, BE&P showed that the M-V rule, which is the foundation of the CAPM, was not always consistent with peoples' choices. From empirical grounds, Levy (2010) suggested the use of ex ante parameters instead of ex post parameters and suggested if researchers employ ex- ante parameters or make small changes in the parameters, the empirical tests on the CAPM will give better results.

Empirical Tests of the CAPM

Sehgal & Balakrishnan (2013) used these values, P/B, P/E, and PSG and took yields on 91-day treasury bills (T-bills) as risk free proxy and applied this model for India. They tested the three factor asset pricing model of Fama-French (FF) in explaining the returns on different portfolios that were constructed based upon company characteristics such as size and value. The data employed was from 1996 to 2010 for 465 companies which formed part of BSE-500 index; the methodology that was adopted was the same as that of Fama-French (1993 and 1996). The result indicated that FF model was a better fit than CAPM for explaining the returns on most of the portfolios that were constructed based on company characteristics; they also found the same result for alternative versions of FF model. Then, they confirmed the presence of strong size and value effects in the Indian stock market and three factor model was proved to be a better descriptor of returns on company characteristics compared with one factor CAPM.

Berger (2011) did a study to prove that the reason for anomalies in the empirical studies of capital asset pricing model (CAPM) is that the ex-post observations were not a good proxy for ex-ante expectations. He doubted on the fact that realized returns are a good proxy for expected future returns. Therefore, he used an alternative specification to proxy for investor expectations, and then tested the CAPM in the context of pricing size and book/market equities and concluded that he obtained a better result. In other words, he asserted that the existing asset pricing models which were based on Markowitz (1952) portfolio selection focused on forward looking expected returns. Moreover, the CAPM that was originally developed by Sharpe (1964) and Lintner (1965) described a relationship between expected future return and risk. Therefore, in order to test the CAPM, researchers must define a measure for expected returns, as true expectations are not observable. Existing empirical studies tend to replace ex-ante expected returns with ex-post observed returns, implicitly assuming that observed returns closely relate to prior expectations.

Borys (2011) in a study in the stock market in the Visegrad countries (the Czech Republic, Hungary, Poland, and Slovakia) showed that CAPM was not to be useful in the case of emerging countries. CAPM was not regarded as a suitable model for asset pricing in emergent countries because, the market in these countries are less liquid. He compared the results with CAPM model and the macroeconomic factor models for these countries and concluded that the factor models can better explain the expected returns in the stock market for Visegrad countries. He specifically emphasized the macro models that used excess market return, industrial production, inflation, money supply, exchange rate variations, and exports. Although the economy has grown in an accelerated proportion in emerging economies, there is more risk involve in the investment in these countries and foreign investors expect to earn a higher returns to invest there. Typically, emerging markets have higher return and higher volatility of stock returns as compared with developed countries. Thus, the explanation of the returns in these markets and the estimate of the cost of equity capital should be different from developed markets, specifically for Visegrad countries. There are other studies that came to the same conclusion (Erb, Harvey, and Viskanta, as cited in Borys, 2011).

Alves (2013) did a comparative analysis of the capital asset pricing model (CAPM) and the Fama and French Model (FFM) for the matter of evaluating investment projects from the point of view of a financial analyst. The focus of the paper was on local, international, and European Monetary Union (EMU) countries and the research question was whether asset size and financial distress premium would have any significance for asset price evaluation by the financial analysts? The research includes a sample of firms from ten countries of the European Monetary Union and he employed FFM model that was an expanded model of CAPM by taking into account size and value factors in addition to the market risk factor of CAPM. He did the research both locally and internationally and tested both traditional CAPM and FFM. They concluded that FFM was preferable in comparison with CAPM for small and high-book to market firm returns.

Febrian & Herwany (2010) applied capital asset pricing model (CAPM) and arbitrage pricing theory (APT) to calculate the excess returns of portfolio of stocks traded on the Jakarta Stock Exchange (JKSE*).* The data were used for three periods of (1992-1997), the crisis period

(1997-2001), and the post-crisis period (2001-2007*)*. The findings showed that CAPM did not single handedly explain portfolio excess returns. The APT however, was able to explain the portfolio excess returns in the observation periods where excess return averages were found to be consistently negative. Moreover, the validity of the existing theories in the recent Asian financial crises years stimulates additional challenges to the transformation in financial science and practice that has had significant advancement in asset pricing approaches. This progress has been proven to be much influenced by the dynamic phenomena of macroeconomic indicators. However, since most of the asset pricing models were constructed based on the observations from macroeconomic data of developed economies, which is more stable and predictable than that of developing economies, performance of the models using developing economies' data would remain questionable. As a whole, they concluded that, specifically in the times of Asian financial crisis, the validity of asset pricing models in the emerging markets is more doubtful. With respect to the macroeconomic data, they noticed that the two variables of exchange rate and spread between the central bank rate and the commercial bank rate were consistently significant in all APT test results. The conclusions indicated that APT was a better model of trading behavior and process in the Indonesian stock market.

Spyroua & Kassimatis (2009) did a study on the time -variation in the value premium and the CAPM in the international market. This study was based on the factor book to market value and compared the value premium for the value versus growth stocks. As a simple definition, value stocks are defined as the stocks with high book to market value and growth stocks are defined as the stocks with low book to market value. Investors invest their excess funds in the growth stocks in order to sell it in the future and gain a profit; but, they purchase value stocks to earn immediate income and collect dividends. Growth stocks belong to the growth companies that the investors expect them to grow in the foreseeable future and these are the companies that typically growing at a faster rate than the overall markets; these companies often allocate most of their current revenue toward further expansion and have lower dividends. Typically, newer companies with innovative products that are expected to make a big impact in the market in the future are amongst growth companies. On the other hand, value stocks belong to undervalued companies that often provide long-term profits; a value stock is an undervalued stock and typically it trades at a price below where it should be, based on its financial status and technical trading indicators.

The time series data were collected for 12 European countries, from July 1981 up to July 1993. The model tested was original CAPM and then, expanded to other characteristics of firms the most important of which that was book-to-market (B/M) ratio. The B/M equity ratio is the criteria by which the value and growth stocks are identified; value stocks typically go with high B/M and growth stocks usually go with low B/M. The effect of knowing about the B/M ratio for a specific stock is that investors consider low B/M as proxy for health and future growth of the company and therefore too much demand for these stocks would push their prices too much and make them overvalued. On the contrary, high B/M resembles value stocks, although, they are stable, but they do not show the sign of growth in the future; therefore, too little demand make them undervalued in the market. In fact this systematic error in predicting future growth of growth and value stocks and the excessive optimistic looking towards growth stocks and the excessive pessimistic approach towards value stocks is one of the main reasons for creating financial imbalances in the security market.

It is generally belied that there is high level of financial distress (Lakonishok et al., 1994) or a high risk premium for high B/M stocks; thus, the investor requires higher return for high B/M companies to compensate for this risk (Fama and French, FF1995). For this reason, Fama and French in FF model, suggested a risk factor to be included in the standard capital asset pricing models (CAPM) to avoid the value anomaly (FF, 1993). In addition, the previous studies on this topic in relation with the international market have indicated the existence of value premium for value stocks. For instance, Fama and French (1998) in a study including US and 12 other international countries found that the return for value stocks was more than return for growth stocks in 12 out of the 13 countries for the period from 1975 to 1995. Similar empirical studies that were conducted in the international markets confirmed a vigorous value premium for value stocks over growth stocks (Liew and Vassalou, 2000). Moreover, the study showed that the value premium seemed to be significantly related to time variable. These findings could have big impact on investors’ decision making in the security market and they can make a precise decision when to buy or sell.

Gabaix (2011) did a study and included a factor named disasterization to the simple CAPM model; disaster was defined as a situation that affects the capital stock and productivity in the economy. He stated that in the disasterization, economy behaves exactly like the original economy, except that GDP, employment, savings, investment, etc. are scaled down. In this situation, asset prices would react differently: The equity premium would be higher in case of inflation, yield curve would be positively sloped, and bond risk premia would become positive. Yalçın & Ersahin (2011) did a study on the conditional (CCAPM) using a sample of common stocks traded on the Istanbul stock exchange from February 1997 to April 2008, their findings were not compatible with CCAPM. The theoretical support for the conditional CAPM was supported by a significant number of empirical studies recognizing the importance of modeling systematic variation in beta. Other empirical testing of CAPM showed emerging markets have more difficulties with CAPM due to lack of complete market integration (Pereiro, as cited in Yalçın & Ersahin 2011). Furthermore, relatively uncertain financial, economic, and institutional environments in emerging economies lead researchers to take into account various other factors such as political risk and stability into account.

Balvers & Huang (2009) did a research on asset pricing in a monetary economy as an extension of the (CAPM) and they included the real money growth as an additional factor into the model. Their argument was that availability of money in the economy would facilitate transactions and therefore, would affect the marginal value of wealth and thereby consumption in the economy. Their approach was based on the utility formulation in Marshall (1992) in which the marginal value of financial returns was determined by the marginal utility of consumption together with the marginal cost of doing transactions, which by itself depends on the liquidity that was available. They postulated that the discount factor that is used for net present value should vary with real consumption growth as well as with real money growth; where real money growth reflects both the effects of nominal money growth and inflation. The argument was that money was held because it lowered the transaction cost of purchasing consumption goods, as modeled in Marshall (1992). However, the model in this study that was based on the Brownian motion assumption in continuous time and conditional normality was claimed to have obtained a specific solution for the asset pricing equation that outperformed other models. As a matter of fact, CAPM originally was based on the marginal utility theory and this model was a return to the origins and was constructed on Marshall utility theory. This is a two-factor asset pricing model which emerges with either market return and real money growth or consumption growth and real money growth as the factors. The authors used the consumption-based asset pricing model of Breedom (1979) in which a consumption growth factor was adequate by itself for pricing any asset and this was based on the fact that the consumption level was a sufficient statistic for capturing marginal utility of any consumer in financial models. They also empirically proved that the inclusion of real-money-growth into the CAPM and C-CAPM has augmented explains the anomalies of previous models. Moreover, they tested and compared their model with eight more models and found out that their two factor model outperformed the alternatives

CAPM and Business decision making

The CAPM is a theoretical model aimed at valuing financial assets in a security market under the assumption that the asset market is in equilibrium. The CAPM may also be used as a decision criterion for corporations, with the argument that an investment is worth undertaking if and only if the investments' expected rate of return is greater than the (cost-based) risk-adjusted cost of capital (Rubinstein, 1973). The use of the CAPM for capital budgeting purposes goes back to the 1960s and 1970s, when various authors developed a theoretical link between this asset pricing model and corporate capital budgeting decisions (Tuttle and Litzenberger, 1968; Rubinstein, 1973; Bierman and Hass, 1973; and Bogue and Roll, 1974). The resulting capital budgeting criterion suggests that, as long as the CAPM assumptions are met, a firm aiming at maximizing share prices should undertake a project if and only if the project's expected internal rate of return exceeded the project's risk-adjusted cost of capital (Magni 2010).

From business point of view, cost of capital is the opportunity cost that an investor forego by investing in a particular project. With this definition, cost of capital should not be considered as expense that is paid out by the investor, it should be considered as the gain as if the capital was invested in other assets with similar characteristics as those that were invested in. Thus, the cost of capital of a firm is equal to the return on its assets and is a constant determined by the market. Moreover, the particular analysis of required rate of return will reflect in how firms evaluate their performance. For instance, cost of capital for a firm is defined as what the firm can achieve by investing in assets of similar characteristics and it is equal to the opportunity cost of capital to the providers of finance to the firm or investors. This is because, firms in order to be able to finance their long-term investments, they should invest in such a way as to meet the return requirements of its providers of capital or investors.

Amongst different sources of long-term capital: Common equity capital, preferred equity and long-term debt, the most difficult to comprehend and calculate, which has been the subject of enormous theoretical and conceptual debates, is the cost of equity capital. The opportunity cost of common equity capital to a company is the annual rate of return that buyers of the common stock of the company require or expect to receive from their investments and is determined by the rate of return that investors can earn by investing in comparable securities in the capital market. Models developed to estimate the cost of common equity capital, can be categorized into two broad groups, accounting-based models and market-based models. Whereas, accounting-based models calculations are based on the historical averages of a firm's own earned returns, the market-based approach deals with the market value of equity capital instead of its book value.

According to Patterson (1995) the accounting approach ignores the idea that the cost of capital is related to the opportunities available to the investors in the capital market. The market-based approach, on the contrary considers the opportunities available to investors in the capital market for estimating the cost of capital. Patterson classified market-based models into two groups: The implicit models under which came the discounted cash flow (DCF) model and its variants, the q-ratio method, and the historical risk premium model; the explicit models under which came the capital asset pricing (CAPM) model and its variants and the arbitrage pricing (APT) model.

Figure 5 below shows classification of models of cost of common equity capital.

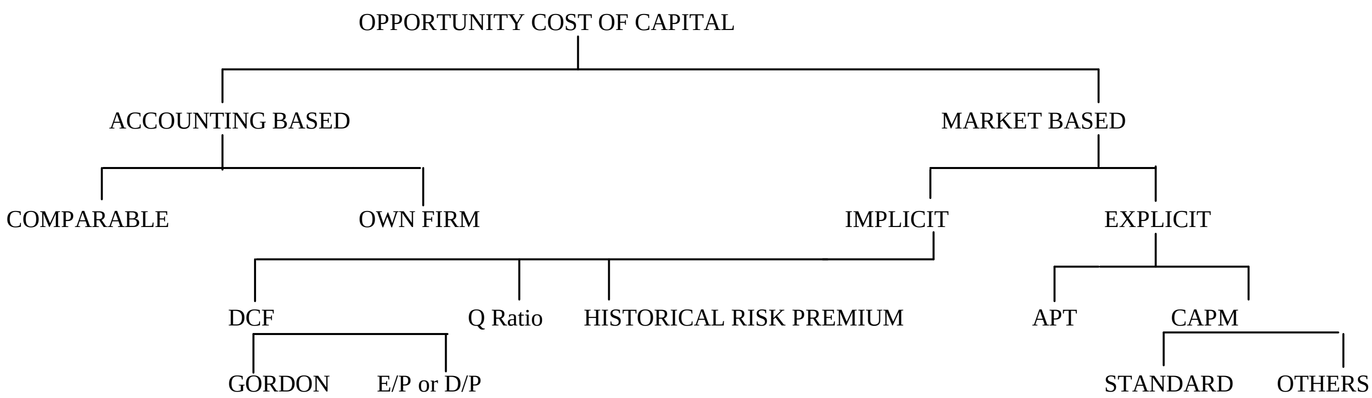

Figure 5. Classification of cost of equity capital models (Source: Patterson, 1995, p. 20).

In other words, the concept of cost of capital, when perceived as the opportunity cost of capital is, (a) a concept that is related to the use of capital not its source, (b) its own return has no bearing on the opportunity cost of capital, and (c) it is the benchmark's rate of return that determines the cost of capital invested in an asset (Patterson, 1995). Thus, we can conclude that from the business point of view, the rate of return is the same thing as the cost of capital of an asset with specific characteristics and the determination of cost of capital is the same as the determination of the annual rate of return that investors require from an asset. Since according to economic theories rate of return is equal to the nominal risk free rate (NRFR) plus some risk premium (RP) that represents the uncertainty or volatility of the expected pay-off from an specific asset, business firms use the same formula to calculate cost of capital for the time span of the investment or with the maturity of the asset.

Typically the calculation is as follows: The nominal risk free rate is determined by the real risk free rate (RRFR) adjusted for the rate of inflation. Real risk free rate is defined as the price that investors charge to exchange current consumption for future consumption when there are no inflation, taxes, and no uncertainty with regard to future investment pay-off. The level of RRFR is determined by investors' subjective time preference and by the availability and nature of investment opportunities in the economy. According to Patterson (1995) although it is difficult to infer the level of RRFR at any point in time, its order of magnitude seems to be around 0-5% per year in most developed economies (p. 3). The relation between real risk free rate RRFR, nominal risk free rate NRFR, and the inflation rate IR is obtained through what is known as the Fisher effect, represented by the following equation:

$$\begin{aligned} NRFR &= (1+RRFR)(1+IR)-1 \approx RRFR+IR \\ E(R) &= NRFR(n)+RP(n) \end{aligned} \qquad (7)$$

Equation 7 indicates that the nominal risk free rate is approximately equal to real risk free rate plus the annual rate of inflation.

The nominal risk-free rate, NRFR, is the rate that can be observed, measured, and estimated in the capital markets and by subtracting the expected inflation rate from NRFR the real risk-free rate, RRFR, can be estimated. The closest approximation to the NRFR in the US economy is the annualized yield to maturity on US government's bills or *zero-coupon* bonds. These instruments are not totally risk free because of fluctuating interest rates in the market and uncertain changing inflation rates; but, if the investors hold to them until the maturity they will receive a sure yield to maturity on their investments. Therefore, depending on the life-span of the investment, the yield to maturity of a government bill or bond with the same maturity as the investment's life-span will be taken as a proxy for the nominal risk-free rate appropriate for calculating the cost of capital of that investment. In this way the term, NRFR (n), will be used to denote the annualized risk-free rate appropriate for evaluating an investment with life-span of *n* years.

The NRFR (n) is the annualized risk-free return required by investors that purchase a government bond with the maturity of *n* years, given their time preferences and inflation expectations. However, if investors' time preferences or their expectations of future inflation changes, then the *expected* pay-off from investment in a particular risk free instrument might not match investor's requirement and depending on the case the investors will sell or buy that instrument until the change in the market price of the instrument causes expected return to equal required return. Thus, when the markets are in equilibrium, that is when the requirements of all investors are satisfied, expected returns equal required returns for all assets. Furthermore, because required return from an asset is the rate investors can earn on assets of similar characteristics, the terms opportunity cost of capital, expected returns, and required return all represent the same thing and are used synonymously. Magni (2010) investigated for a particular version of the net present value (NPV) which makes use of the Capital Asset Pricing Model (CAPM) for computing the discount rate. He proposed the concept of net future value (NFV) and the relation between residual income and NFV. He showed that four decision rules can validly be deducted from the CAPM: the disequilibrium NPV, the equilibrium NFV, the equilibrium NPV, and the disequilibrium NFV; all of these may be interchangeably used for decision making

Conclusion

Asset pricing models have been the center of attention both in the academia and among financial practitioners. A precise evaluation of investment projects is a matter of great importance for academia, practitioners, society as a whole, and Governments; assessments of capital costs, have a profound future impact on taxes and sovereign debts paid by citizens. The emergence of modern finance is based on mainly the Mean-Variance (M-V) efficiency analysis of Markowitz's (1952) and Sharpe (1964) and Lintner's (1965) capital asset pricing model (CAPM). Although Markowitz portfolio selection was not by itself a theory of equilibrium asset prices, it formed the basis of CAPM. CAPM extended Markowitz model by introducing a risk-free asset into the model which investors could add that in their portfolio of risky assets. Thus, investors can lend and borrow at the fixed risk-free rate in the risk free asset market. Then, CAPM concluded that there existed one specific portfolio for every investor that could serve as the optimal portfolio and consists of all risky assets weighted by their market capitalization and labeled the market portfolio. From here, the famous expected return-beta relationship of CAPM was deduced according to which the expected excess return of any security was linearly related to the beta of the security and the expected excess return on the market portfolio. The expected return-beta relationship was the basis of equilibrium asset prices in the capital market and if any deviation from equilibrium prices occur in the market the mean variance efficient investors acting on the basis of expected return-beta relationship will readjust their portfolios and restore equilibrium prices.

Since its inception, CAPM of Sharpe (1964) and Lintner (1965) was considered the base model for hundreds of academic studies and financial practitioners who frequently used the Sharpe Ratio which relied on the M-V model and use beta which was derived from the CAPM as the risk index. Although, CAPM has remained a benchmark model of asset pricing in the academic literature, yet it has been under severe and ongoing theoretical and empirical attacks. Most of the early attacks emerged from numerous empirical studies in finance and economics, which revealed that the CAPM did not fit empirical asset pricing well (Fama & French 2004); also, the use of constant betas in unconditional CAPM was criticized (Fama and French, 1992, 1993, 1996). All studies referred to the poor performance of CAPM in explaining certain irregularities in the asset pricing applications; what is known as anomalies of CAPM.

However, despite all these criticisms, almost all researchers approved of the basic theme of CAPM and proof of all anomalies has improved the original model in different dimensions. The issues of size effect or the outperformance of small firms, the outperformance of the firms with high Book-to-Market (B/M) value or the ‘B/M’ effect, and the ‘momentum’ effect or the outperformance of firms with relatively high returns in the past years that was discussed and analyzed later by researchers has developed the original CAPM model. Quite a few studies documented a significant relation between stock returns and various firm characteristics such as size, book-to-market (B/M) equity, and leverage and earnings-price ratios (Banz, 1981; Basu, 1983; Bhandari, 1988). In addition, the inclusion of time variation in CAPM betas for B/M and momentum portfolios has improved the applicability of CAPM model of asset pricing over both the conditional and unconditional CAPM (Abdymomunova & Morleyb, 2011).

Recently, behavioral economists and psychologists cast doubt on the validity of expected utility theory (EUT) that is the foundation for CAPM analysis and questioned the validity of the M-V rule and the CAPM which are derived within the EUT framework. The core of criticism is that a typical investor is not always a rational as described by EUT economists (Kahneman and Tversky (T&K) (1979, 1992). This would make EUT invalid, and hence, all models that are derived in the EUT framework such as the M-V efficiency analysis and the CAPM, are also questionable. Then, Kahneman and Tversky suggested the prospect theory (PT) and, later on, a modified version of PT, called cumulative prospect theory (CPT), as an alternative for EUT.

The analysis of asset pricing and its understanding has a lot of implications for investors in the stock market. Investors buy undervalued stocks and sell overvalued stocks every day and making correct decisions makes a difference for gain or loss for investors and on a macro level help or hurt the overall economic stability. It is empirically tested that investors react differently in the cycles of economy; that is, time is a big element in decision making by investors (Gabaix, 2011; Balvers & Huang (2009); Spyroua & Kassimatis, 2009). The concept of value premium differentiation between the good and bad states of economy in regards with both value and growth stocks was proved that high B/M stocks perform worse in the downturn than upturn (Spyroua & Kassimatis, 2009). In this study it was evidenced that the perception of investors was different towards valuation of growth and value stocks in the boom market and the down market. For example, growth stocks that typically have a better prospect of growth in the near future were perceived as less risky during boom markets and more risky during down markets by investors. It is notable that in the late 1990s, the price of internet, technology industries rose significantly compared to more traditional stocks. On the contrary, in the down market, value stocks were perceived as less risky than growth stocks; for instance, after 2000, internet stocks fell more rapidly compared with value stock. It is also evidenced that it is the nature of human beings that over exacerbate their forecast in the business cycles. In other words, investors are excessively optimistic about growth and excessively pessimistic about value stocks during boom market and make systematic errors in predicting future earnings growth; contrary is true in the down market. Therefore, as a result of these findings, it is evidenced that the systematic risk seemed to change between good and bad economic conditions (Spyroua & Kassimatis, 2009).

Thus, although simple models make it easier for researchers, when it comes to the credibility of results, they seemed to be deficient. Financial crisis of 2008 that was the largest hit to the US and world economy since great depression of 1929, proved that macroeconomic conditions cannot be ignored in micro financial decisions. The intertwining reaction between macroeconomic factors and micro decisions are a topic that has to be given more attention in the economic- financial research. Inclusion of time, business cycles and disasterization into asset pricing models are an effective way of solving the problem. However, what is needed is a comprehensive asset pricing model that encompasses all the relevant macroeconomic factors with the goal of a unified model of macro-finance asset pricing model.

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